

# Overview of the IMPECC system for $4\pi\beta\text{-}\gamma$ coincidence counting at LNHB

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# Context

- IMPECC: acronym for « Instrument de Mesure de Périodes Communes de Calme »
  - First coincidence system based on the live time technique developed at LNHB in 1976
  
- Development following the intercomparison of increasing activity sources of  $^{60}\text{Co}$  (1975/1976)
  - Need for more accurate calculation of accidental coincidences
  - Development of the Cox and Isham formulas (1977)
    - D. Smith, A. Williams and M.J. Woods, Intercomparison of high-count-rate  $^{60}\text{Co}$  sources, Rapport BIPM-77/7, 65 p. (1977). A French translation "Comparaison internationale de sources de  $^{60}\text{Co}$  à taux de comptage élevé" appeared in CCEMRI, Section II, 4<sup>e</sup> réunion-1977, Annexe R(II)3, BIPM (1977)
  
- IMPECC inspired from a paper of Gandy (1963)
  - Correction for accidental coincidences based on the use of the live time technique
  
- First development of the IMPECC electronics by J. Bouchard (1976)
  
- Development of specific formulas for accidental coincidences for high-counting-rate sources by B. Chauvenet (1986)



## Main aspects of Gandy's paper (1963)

- Dead-times defined by the time-over-threshold of pulses in the  $\beta$ - and  $\gamma$ -channels
- Dead-time in the coincidence channel: logical OR operator of dead-times in the  $\beta$ - and  $\gamma$ -channels
- Implementation of the live time technique in the coincidence,  $\beta$ - and  $\gamma$ -channels
  - Probability for a channel to be in live time:  $P = T_a/T$  ( $T_a$ : active time,  $T$ : total time)
  - Measured counting rate in a channel:  $N' = N.P$  ( $N$ : true counting rate)
- Instrumental correction for accidental coincidences
  - Measured coincidence counting rate:  $N'_c = N_c.P_c + N_\beta.(P_\beta - P_c) + N_\gamma.(P_\gamma - P_c)$
  - $N_\beta.(P_\beta - P_c)$ : counting rate of accidental coincidences triggered by a gamma pulse
  - $N_c$ : counting rate of true coincidences

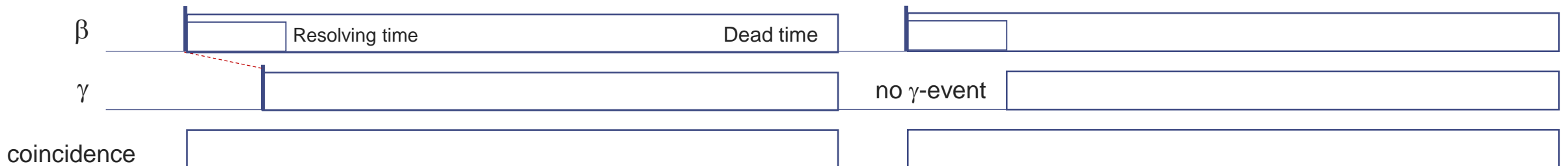
Mesure Absolue de l'Activité des Radio-nuclides par la Méthode des Coïncidences  $\beta$ - $\gamma$  Etude d'une Méthode de Correction Automatique des Erreurs Instrumentales

A. GANDY  
Fondation Curie, Paris, France

Problem  
Only exact in the case  
of true coincidences  
without time jitter

# Electronics of IMPECC (J. Bouchard)

- Interest of the Gandy's approach:
  - the live time technique can be applied whatever the type of the dead time used
  - the calculation of accidental coincidences can be made using the live time measurements
- Both  $\beta$ - and  $\gamma$ -channels divided into two specialized channels:
  - Fast channel starts the pulse processing: reconstructible dead time, resolving time, pileup rejector, ...
  - Slow channel for amplitude analysis (based on Lecroy ADC and Memory modules)
- Acquisition a 3D amplitude spectrum allowing the storage of coincident and non coincident events
- Implementation of a common cumulative dead times in the coincidence,  $\beta$ - and  $\gamma$ -channels



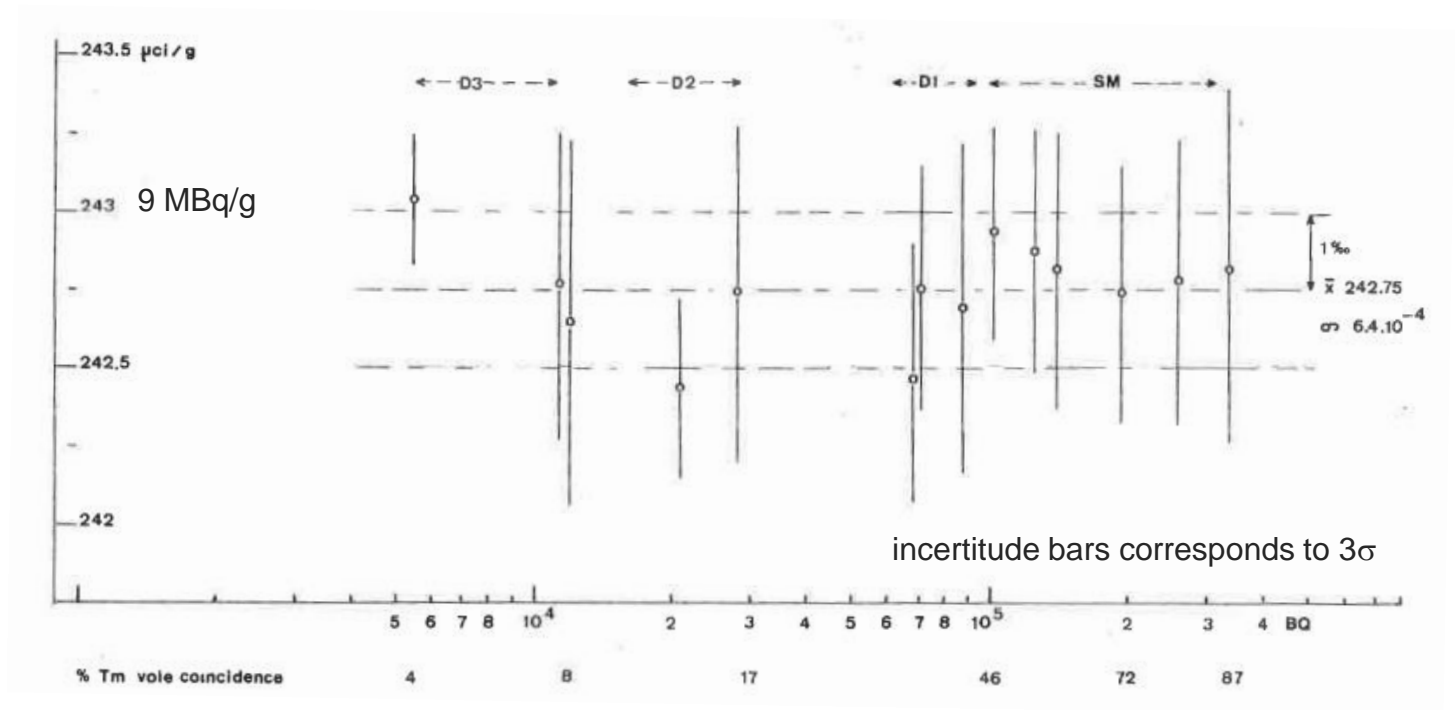
# First experimental results (1977)

- Modified formula for accidental coincidences

- Measured coincidence counting rate:  $N'_c = N_c \cdot P_c + N_\beta \cdot (P_\beta - P_c - C'_0 \delta) + N_\gamma \cdot (P_\gamma - P_c - C'_0 \delta)$ 
  - Measured coincidence counting:  $C'_0$
  - Quadratic mean of time jitter:  $\delta$

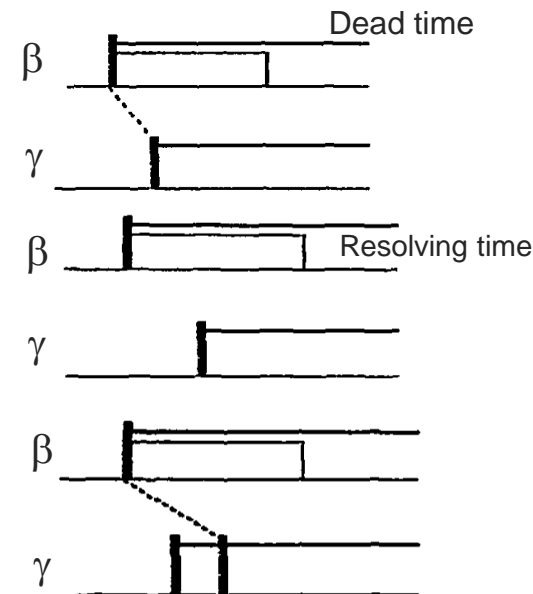
- Measurements of increasing activity sources

- Co-60
- Activity between 5 kBq and 335 kBq
- Proportional counter in the  $\beta$ -channel
- Ge-Li detector in the  $\gamma$ -channel



# Development of exact correction formulae (B. Chauvenet)

- Poisson process only valid in the coincidence channel regarding the application of the live time technique
  - Because of the time jitter between the  $\beta$ - and  $\gamma$ -channels:  $N'_\beta \neq N_\beta \cdot P_\beta$  and  $N'_\gamma \neq N_\gamma \cdot P_\gamma$
  - Only the coincidence channel follows a Poisson process because the dead time period is always triggered by the first pulses of coincidence pairs
  - The measured counting rate in the coincidence channel:
    - ✓  $N'_\beta + N'_\gamma - N'_c = P_c \cdot (N_\beta + N_\gamma - N_c)$
    - ✓  $N'_c = N_c \cdot P_c + (N'_\beta - P_c N_\beta) + (N'_\gamma - P_c N_\gamma)$
- Two relations corresponding to measured coincidence rates started by a  $\beta$ -pulse ( $N'_{\beta\gamma}$ ) or started by a  $\gamma$ -pulse ( $N'_{\gamma\beta}$ )
  - ✓  $N'_\gamma - N'_{\beta\gamma} = P_c \cdot (N_\gamma - P_{\beta\gamma} N_c)$  ( $P_{\beta\gamma}$ : probability to measure a coincidence started by a  $\beta$ -pulse)
  - ✓  $N'_\beta - N'_{\gamma\beta} = P_c \cdot (N_\beta - P_{\gamma\beta} N_c)$  ( $P_{\gamma\beta}$ : probability to measure a coincidence started by a  $\gamma$ -pulse)
- Regarding the measured coincidence rate started by a  $\beta$ -pulse
  - True coincidences (triggered by a true coincidence event):
  - Accidental coincidences of type I (triggered by a non-coincident event):
  - Accidental coincidences of type II (triggered by a true coincidence event):
    - ✓ True coincidence lost due to the time jitter



# Calculation of $N'_{\beta\gamma}$

- Definitions

- Probability density to count a  $\gamma$ -pulse between  $t$  and  $t+dt$  after the  $\beta$ -pulse (true coincidence):  $f_{\beta\gamma}$
- Probability that the delay is greater than  $t$  (true coincidence):  $F_{\beta\gamma}(t)$  (survival function)
- Probability of non-counting a  $\gamma$ -pulse from another disintegration until  $t$ :  $I_{\beta\gamma}(t)$

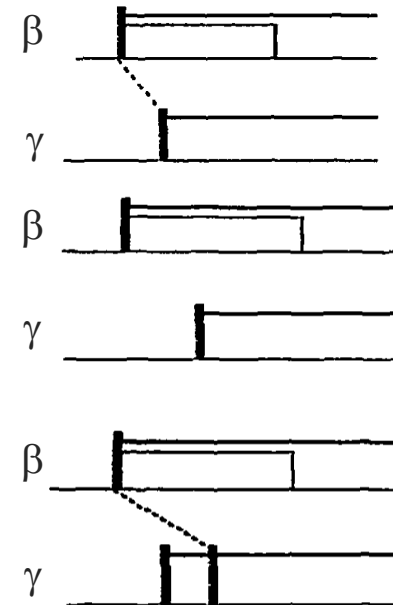
$$\checkmark I_{\beta\gamma}(t) = \exp \int_0^t - (N_\gamma - N_c P_{\beta\gamma} F_{\beta\gamma}(u)) du$$

- Expressions of the measured coincidence counting rate  $n'_{\beta\gamma}(t)dt$  between  $t$  and  $t+dt$  for the three types of coincidence counting:

- True coincidences:  $P_c \cdot P_{\beta\gamma} \cdot N_c \cdot I_{\beta\gamma}(t) f_{\beta\gamma} dt$

- Accidental coincidences (type I):  $P_c \cdot (N_\beta - N_c) \cdot (N_\gamma - N_c P_{\beta\gamma} F_{\beta\gamma}(t)) I_{\beta\gamma}(t) dt$

- Accidental coincidences (type II):  $P_c \cdot P_{\beta\gamma} \cdot N_c \cdot F_{\beta\gamma}(t) \cdot (N_\gamma - N_c P_{\beta\gamma} F_{\beta\gamma}(t)) \cdot I_{\beta\gamma}(t) dt$



## Exact formulae for $N'_{\beta\gamma}$ and $N'_{\gamma\beta}$

- Integrating the measured coincidence counting rate  $n'_{\beta\gamma}(t)$  over the resolving time  $t_r$  gives:
  - $$N'_{\beta\gamma} = P_c \cdot N_c \cdot P_{\beta\gamma} + P_c \cdot (N_\beta - N_c) \left( 1 - \exp(N_c \cdot P_{\beta\gamma} \cdot t_{\beta\gamma} - N_\gamma \cdot t_r) \right)$$
    - ✓ With the mean value of the gamma delay  $t_{\beta\gamma} = \int_0^{t_r} F_{\beta\gamma}(u) du$
    - ✓ The first term of  $N'_{\beta\gamma}$  represents the measured counting rate of true coincidences and type II accidental coincidences
    - ✓ The second term represents the accidental coincidences of type I
  - Similarly for the measured counting rates for coincidences triggered by  $\gamma$ -pulses:
    - $$N'_{\gamma\beta} = P_c \cdot N_c \cdot P_{\gamma\beta} + P_c \cdot (N_\gamma - N_c) \left( 1 - \exp(N_c \cdot P_{\gamma\beta} \cdot t_{\gamma\beta} - N_\beta \cdot t_r) \right)$$
- According to B. Chauvenet, these expressions and the following are the most rigorous formulas obtainable
  - $$N'_c = N_c \cdot P_c + (N'_\beta - P_c N_\beta) + (N'_\gamma - P_c N_\gamma)$$



# Approximated relations

- Chauvenet proposes a way to calculate the expressions of  $N_\beta$  and  $N_\gamma$  using the following relations:

- $P_\gamma - P_c = (N'_\beta - N'_c) \cdot t_r + \int_0^{t_r} n'_{\beta\gamma} \cdot t \cdot dt$  (corresponding to a  $\beta$ -pulse first)
- $P_\beta - P_c = (N'_\gamma - N'_c) \cdot t_r + \int_0^{t_r} n'_{\gamma\beta} \cdot t \cdot dt$  (corresponding to a  $\gamma$ -pulse first)

- With the following approximations:

- $F_{\beta\gamma} = 1$  when  $t \leq t_{\beta\gamma}$  and  $F_{\beta\gamma} = 0$  otherwise
- $F_{\gamma\beta} = 1$  when  $t \leq t_{\gamma\beta}$  and  $F_{\gamma\beta} = 0$  otherwise

- $$N_\gamma = \frac{(N'_\beta + N'_\gamma - N'_c) \cdot \exp\left(\frac{(N'_{\beta\gamma} - N'_\gamma) t_{\beta\gamma}}{P_c}\right) - (N'_\beta - N'_c)}{P_\gamma - P_c \cdot \left(1 - \exp\left(\frac{(N'_{\beta\gamma} - N'_\gamma) t_{\beta\gamma}}{P_c}\right)\right) \cdot (N'_\beta + N'_\gamma - N'_c) / (N'_\gamma - N'_{\beta\gamma})}$$
- $$N_\beta = \frac{(N'_\beta + N'_\gamma - N'_c) \cdot \exp\left(\frac{(N'_{\gamma\beta} - N'_\beta) t_{\gamma\beta}}{P_c}\right) - (N'_\gamma - N'_c)}{P_\beta - P_c \cdot \left(1 - \exp\left(\frac{(N'_{\gamma\beta} - N'_\beta) t_{\gamma\beta}}{P_c}\right)\right) \cdot (N'_\beta + N'_\gamma - N'_c) / (N'_\beta - N'_{\gamma\beta})}$$

- These expressions can be used when the delay distributions are narrow (a few hundred of ns) and  $t_{\beta\gamma}$  and  $t_{\gamma\beta}$  are known
- When the mean delays converge to 0,  $N'_\beta = N_\beta \cdot P_\beta$  and  $N'_\gamma = N_\gamma \cdot P_\gamma$  (Gandy's formulas)

# Unfolding method

- When the delay distributions are wider and unknown, an unfolding method is applied from the measurement of the distribution of the gamma delay, using:
  - a delay imposed in the  $\gamma$ -channel to obtain  $P_{\gamma\beta} = 0$  and  $P_{\beta\gamma} = 1$ 
    - ✓ leading to simplified expressions:  $N'_\beta = N_\beta \cdot P_\beta$  and  $N'_\gamma - N'_{\beta\gamma} = P_c \cdot (N_\gamma - N_c)$
  - a time-to-amplitude converter (TAC)
- The TAC spectrum gives the experimental distribution of the gamma delay  $n'_{\beta\gamma}(t)$ 
  - with  $h(i) = N_c f_{\beta\gamma}(t)$  with  $t$  the delay corresponding to channel  $i$
  - using the relation  $n'_{\beta\gamma}(t)$ , it is possible to calculate  $h$  for each channel with a recurrent procedure

$$h(i)w = \frac{1}{P_c} c(i) \exp\left( (N_\gamma - N_c)w(i - 0.5) + \sum_{j=1}^{i-1} \sum_{k=1}^j h(k)w^2 \right) - \left( N_\beta - \sum_{j=1}^{i-1} h(j)w \right) \left( N_\gamma - N_c + \sum_{j=1}^{i-1} h(j)w \right) w.$$

- With the  $h$  function, we calculate:
  - the true coincidence rate  $N_c = \sum h(i) \cdot w$  ( $w$ : channel width)
  - the mean delay  $t_{\beta\gamma} = w \sum i h(i) / \sum h(i)$
  - the distributions for the three types of coincidences

# Monte Carlo simulations (Sr-85)

- Validation of the unfolding method by Monte Carlo simulations of activity measurements of  $^{85}\text{Sr}$ 
  - Metastable state of  $1\ \mu\text{s}$
  - Delay in the  $\gamma$ -channel of  $4\ \mu\text{s}$
  - Resolving time equal to  $18\ \mu\text{s}$
  - Activity  $20\ \text{kBq}$  ( $\varepsilon_{\beta} = 0.9$ ;  $\varepsilon_{\gamma} = 0.2$ )

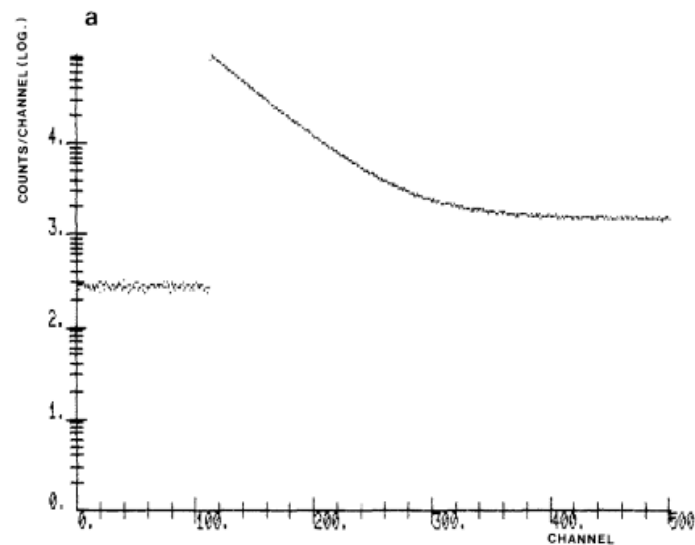
## PROPERTIES OF A $4\pi\beta$ - $\gamma$ COINCIDENCE SYSTEM WITH A CUMULATIVE DEAD-TIME CIRCUIT

B. CHAUVENET, J. BOUCHARD and R. VATIN

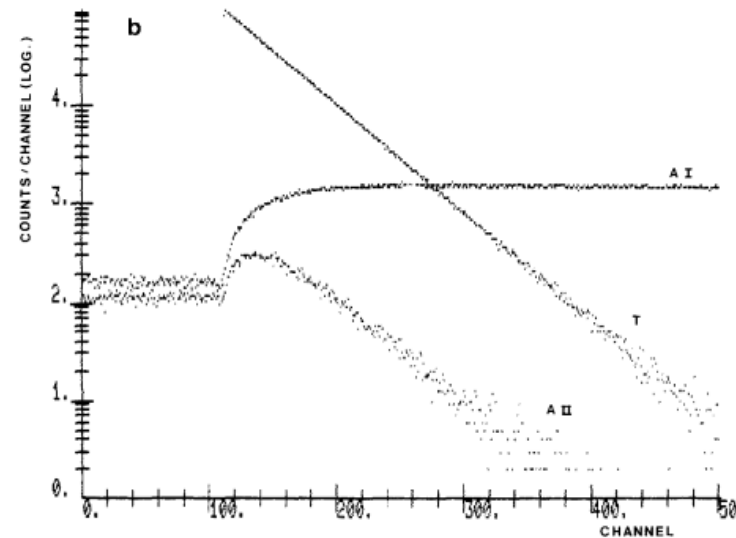
LMRI, CEN Saclay, BP 21, 91190 Gif-sur-Yvette, France

Received 25 July 1985 and in revised form 17 September 1985

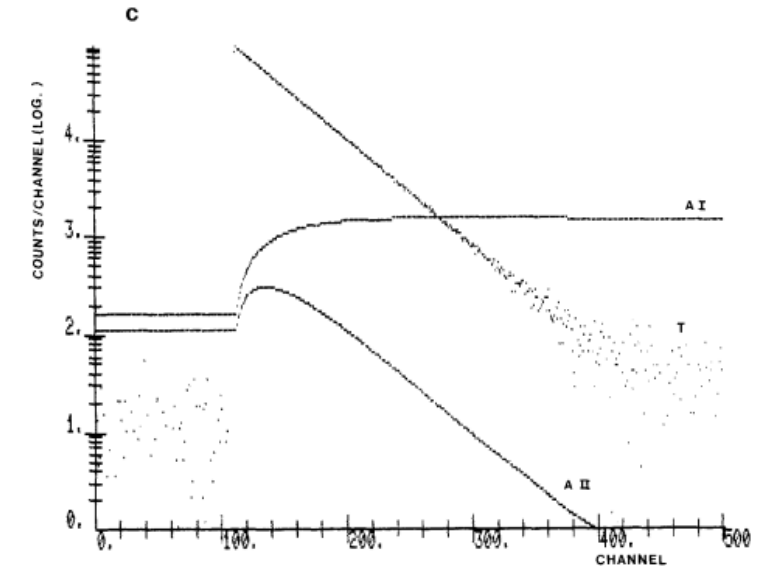
SIMULATION STRONTIUM 85



SIMULATION SR 85: STORED SPECTRA

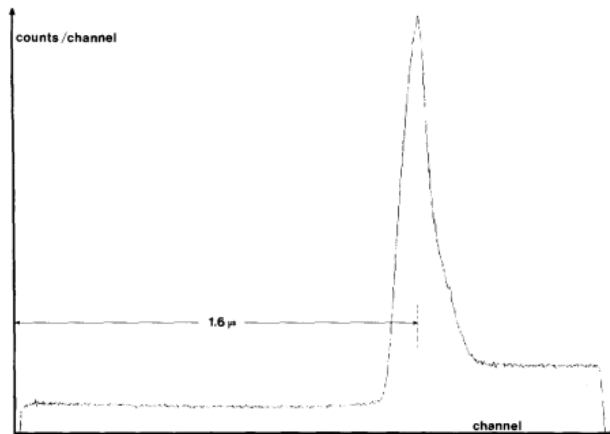


SIMULATION SR 85: UNFOLDED SPECTRA



# Measurements of high-count-rate sources of $^{60}\text{Co}$

- Measurement conditions
  - Proportional counter in the  $\beta$ -channel and a GeHP detector in the  $\beta$ -channel
  - Delay in the  $\gamma$ -channel of  $1.6 \mu\text{s}$
  - Resolving time equal to  $2.5 \mu\text{s}$
  - Activity between  $8 \text{ kBq}$  and  $1.3 \text{ MBq}$  (96% dead time ratio)
  - Coincidence rates (true and accidental) calculated using the unfolding method

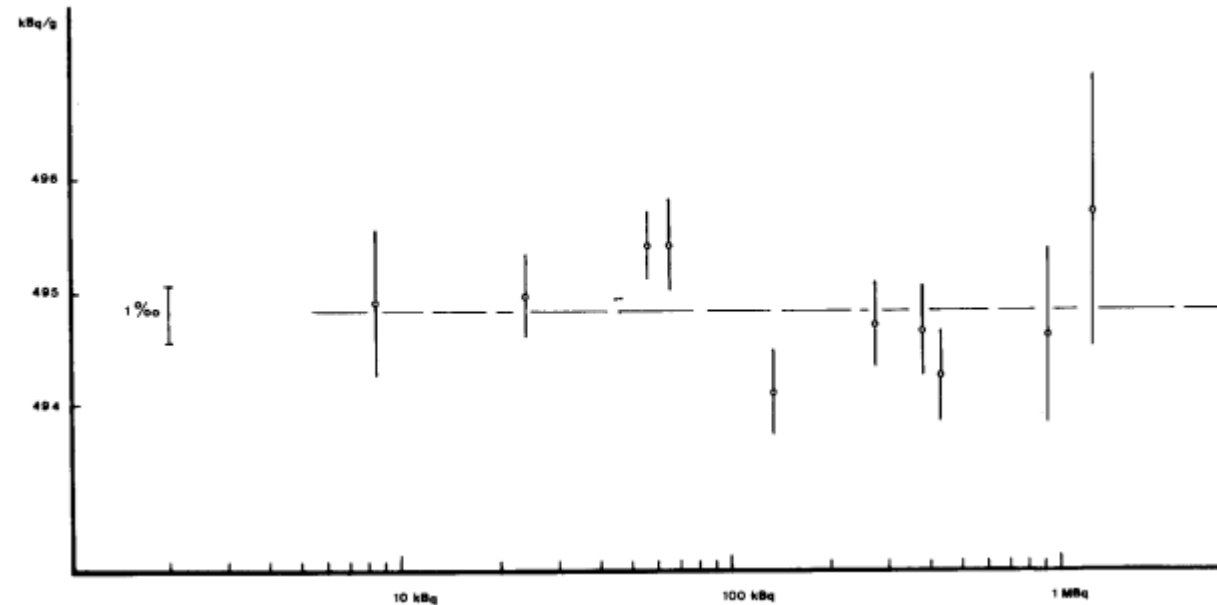


## MEASUREMENT OF HIGH-ACTIVITY SOURCES WITH A $4\pi\beta$ - $\gamma$ COINCIDENCE SYSTEM

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# Conclusion

- Inspired from a paper of Gandy, the IMPECC system represents a different approach for  $4\pi\beta\text{-}\gamma$  coincidence counting based on the implementation of common cumulative dead times in the coincidence,  $\beta$ - and  $\gamma$ -channels and the use of the live time technique
- Rigorous formulas were developed to correct for classical corrections: dead time, time jitter and accidental coincidences
- Good results obtained with high-count-rate sources are made possible by the dead time circuit realizing clear, unambiguous periods of live time, which can be properly taken into account in the calculations.
- A variant of the selective sampling method developed by Müller was also tested
- All the functionalities needed for the implementation of the IMPECC can be carried out using a fast digitizer

# Bibliography

Nuclear Instruments and Methods in Physics Research A243 (1986) 539–548  
North-Holland, Amsterdam

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## PROPERTIES OF A $4\pi\beta\text{-}\gamma$ COINCIDENCE SYSTEM WITH A CUMULATIVE DEAD-TIME CIRCUIT

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Received 25 July 1985 and in revised form 17 September 1985

A coincidence measuring system working with cumulative dead times is presented. The principles of counting for beta, gamma and coincident pulses are developed and the formulae to be applied are derived. A method inspired from selective sampling is also described. All the formulae have been checked with Monte Carlo simulations, showing the ability of accurate measurements even at high count rates.

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Nuclear Instruments and Methods in Physics Research A259 (1987) 550–556  
North-Holland, Amsterdam

## MEASUREMENT OF HIGH-ACTIVITY SOURCES WITH A $4\pi\beta\text{-}\gamma$ COINCIDENCE SYSTEM

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A  $4\pi\beta\text{-}\gamma$  coincidence system (IMPECC) working with cumulative dead times is described. The main features of the electronics of that system are presented, and its capability of measuring accurately sources of very high activity is demonstrated experimentally with a set of  $^{60}\text{Co}$  sources. Results are coherent, and no bias is observed with increasing activities, even for the source of 1.3 MBq which was measured with a relative uncertainty that did not exceed 0.25%. A method derived from selective sampling was also tested successfully with the sources of highest activity.

